Il moto browniano e oltre

Francesco Tampieri

ISAC CNR, Bologna, Italy

f.tampieri@isac.cnr.it

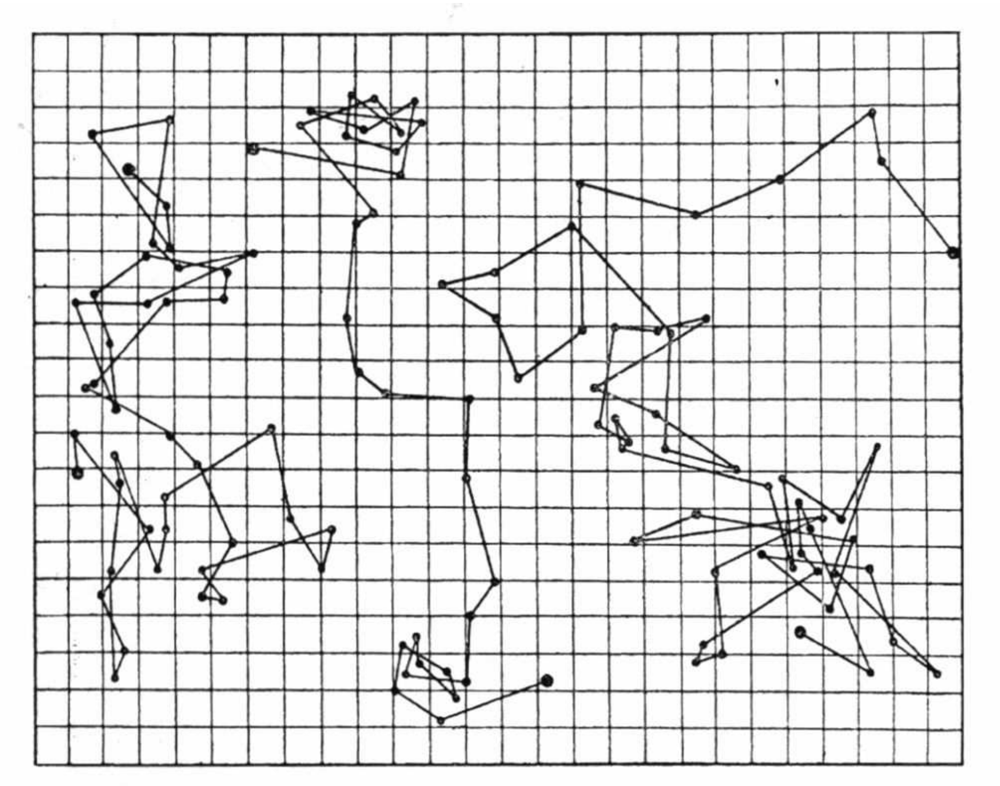
The true logic for this world is the calculus of probabilities (J.C.Maxwell)

Due parole di storia, da Haw (2005)

- Robert Brown (1827) osservazione del moto browniano: di cosa si tratta ....
- Meccanica reversibile e termodinamica, verso la fine del XIX secolo
- La meccanica statistica (Boltzmann)
- Albert Einstein (1905) articolo (Einstein, 1905)
- Paul Langevin (1908) equazione stocastica per il moto browniano
- Jean Perrin (1908, 1913) misura in laboratorio del coeff. di diffusione browniano

E oggi?

Una simulazione



Simulazione di tre particelle browniane, rappresentate a intervalli di 30 s. Ogni divisione del reticolo corrisponde a 3.125 μm. Da DiGiacomo (2005).

Focalizziamo qualche idea

- connessione macroscopico-microscopico: attrito del fluido (legge di Stokes, descrizione del fluido come un continuo) vs equipartizione dell'energia cinetica delle molecole
- calcolo stocastico: non possiamo dare una legge deterministica per ciascuna particella (non possiamo neppure stimare la velocita' media da spazio diviso tempo) ma possiamo dare leggi deterministiche per i momenti della distribuzione

Un poco di matematica elementare

media e varianza

(1)
$$\langle x_i \rangle = \frac{1}{N} \sum_{n=1}^{N} x_i^{(n)}$$

(2)
$$\langle x_i^2 \rangle = \frac{1}{N} \sum_{n=1}^{N} \left(x_i^{(n)} \right)^2$$

(3)
$$\langle x_i^2 \rangle = \langle x_i \rangle^2 + \frac{1}{N} \sum_{n=1}^{N} \left(x_i^{(n)} - \langle x_i \rangle \right)^2$$

il cammino del marinaio ubriaco

(4) $$x(t + \Delta t) = x(t) + \Delta x$$

(5) $$\langle x(t + \Delta t)\rangle = \langle x(t)\rangle + \langle \Delta x\rangle = \langle x(t)\rangle$$

(6) $$\langle x(t + \Delta t)^2\rangle = \langle x(t)^2\rangle + \langle (\Delta x)^2\rangle + 2\langle x(t)\Delta x\rangle = \langle x(t)^2\rangle + \langle (\Delta x)^2\rangle$$

(7) $$\langle x(t)^2\rangle = \alpha t$$

la derivazione formalizzata

(8) $$\mathbf{F} = m\mathbf{a}$$

- forza casuale dovuta agli urti delle molecole $\mathbf{X}$
- attrito del fluido, che a bassa velocita' e' dato dalla formula di Stokes $\mathbf{F}_A = -6\pi\rho\nu a\mathbf{v}$. Aria: $\rho = 1.2 kgm^{-3}$, $\nu = 1.5\ 10^{-5}\ m^2 s^{-1}$. Acqua: $\rho = 10^3\ kgm^{-3}$, $\nu = 10^{-6}\ m^2 s^{-1}$

Quindi:

(9) $$m\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} = -6\pi\rho\nu a\mathbf{v} + \mathbf{X}$$

$\mathbf{v} = \mathrm{d}\mathbf{x}/\,\mathrm{d}t$

l’energia cinetica della particella

Particella in equilibrio termodinamico col fluido in cui e’ immersa:

(10) $$\frac{1}{2}m\langle v_i^2\rangle = \frac{1}{2}kT = \frac{1}{2}\frac{R}{N}T$$

- T: temperatura assoluta, in gradi Kelvin K
- R: costante dei gas $8.31\ JK^{-1}mole^{-1}$
- N: numero di Avogadro 610^{23}

la soluzione

Mediando su molte traiettorie, e ricordando che la correlazione tra forza casuale e spostamento e' nulla: $\langle X_i x_i \rangle = 0$, si ottiene

$$\frac{1}{2}\frac{\mathrm{d}^2\langle x_i^2\rangle}{\mathrm{d}t^2} = -\frac{3\pi\rho\nu a}{m}\frac{\mathrm{d}\langle x_i^2\rangle}{\mathrm{d}t} + \langle v_i^2\rangle \tag{11}$$

$$\frac{\mathrm{d}\langle x_i^2\rangle}{\mathrm{d}t} = \frac{kT}{3\pi\rho\nu a}\left[1 - \exp\left(-\frac{6\pi\rho\nu a}{m}t\right)\right] \tag{12}$$

Tempo di scala $T_p = m/(6\pi\rho\nu a)$.

capiamo la soluzione

- $t \ll T_p$:

(13)
$$\frac{\mathrm{d}\langle x_i^2\rangle}{\mathrm{d}t} \simeq 2\frac{kT}{m}t$$

(14)
$$\langle x_i^2\rangle \simeq \frac{kT}{m}t^2 \equiv \langle v_i^2\rangle t^2$$

- $t \gg T_p$:

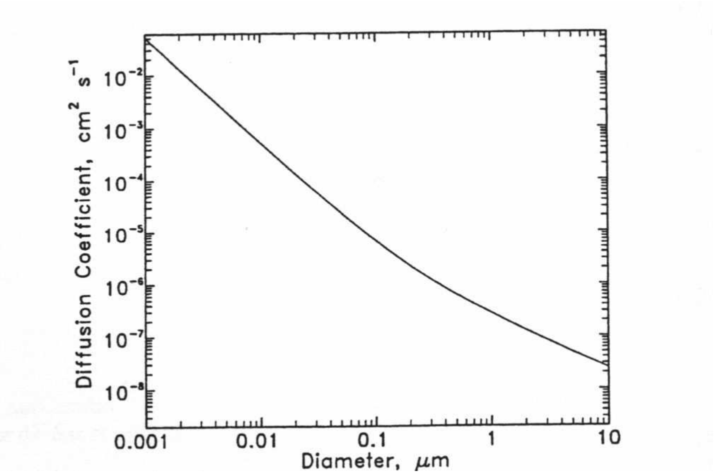
(15)
$$\frac{\mathrm{d}\langle x_i^2\rangle}{\mathrm{d}t} \simeq \frac{kT}{3\pi\rho\nu a}$$

(16)
$$\langle x_i^2\rangle \simeq \frac{kT}{3\pi\rho\nu a}t = 2\langle v_i^2\rangle T_p t$$

coefficiente di diffusione

(cfr. Isichenko, 1992, pag. 1003):

(17) $$D \equiv \lim_{t\to\infty} \frac{\langle x_i^2 \rangle}{2t} = \frac{kT}{6\pi\rho\nu a} = \langle v_i^2 \rangle T_p$$



coefficiente di diffusione browniano per particelle di aerosol (densita' pari all'acqua) in aria. Nota che il coeff. di diffusione del calore in aria e' $\kappa = 0.21\,\mathrm{cm}^2\mathrm{s}^{-1}$ a temperatura ambiente.

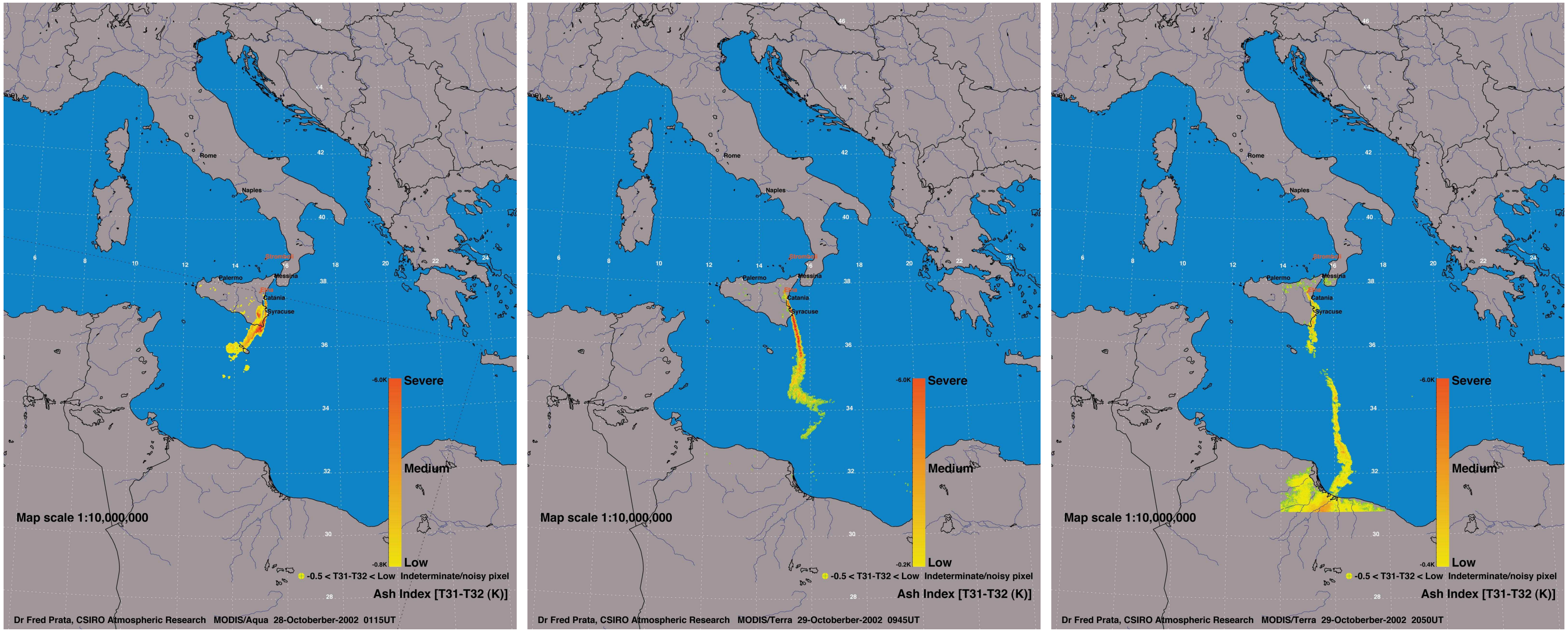
moto browniano oggi: un'eredita'

l'importanza della descrizione stocastica dei fenomeni:

- biologia, nanotecnologie, ... applicazioni alle scienze economiche, sociali, modelli di traffico, del tempo e del clima.
- qualche esempio tra le cose che conosco: la meccanica statistica dei fluidi, ovvero, la diffusione degli inquinanti in atmosfera

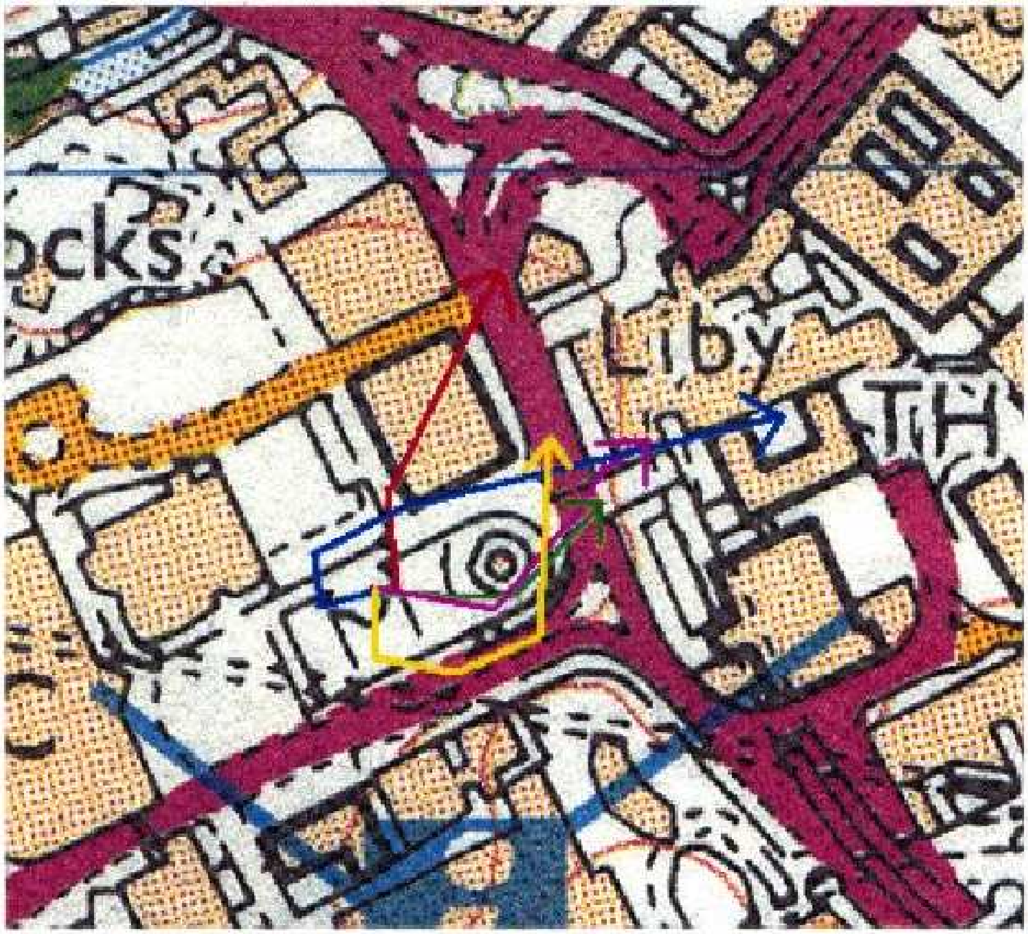
Volcanic dust, from Tiesi et al. (2006)

Thermal image of the Etna plume, 2002. The ash cloud behaves as a plume in a turbulent flow.

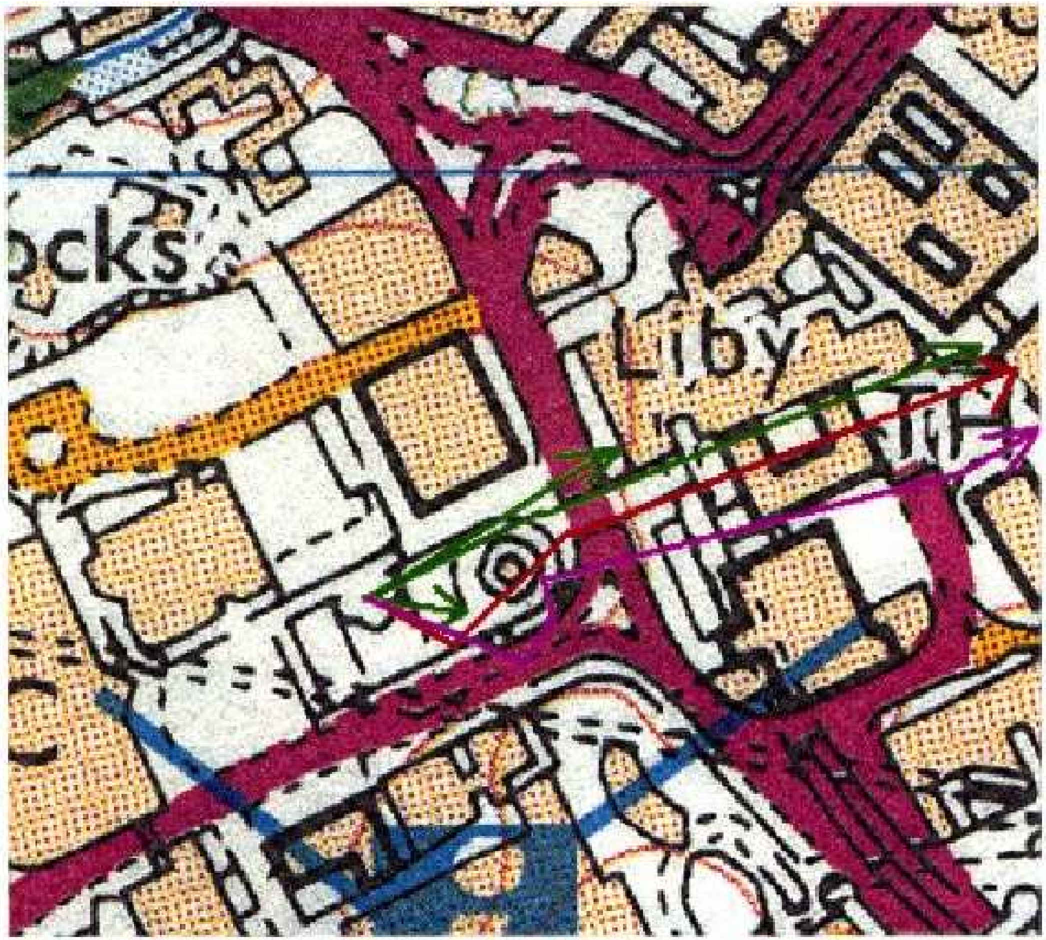


La dispersione in ambiente urbano

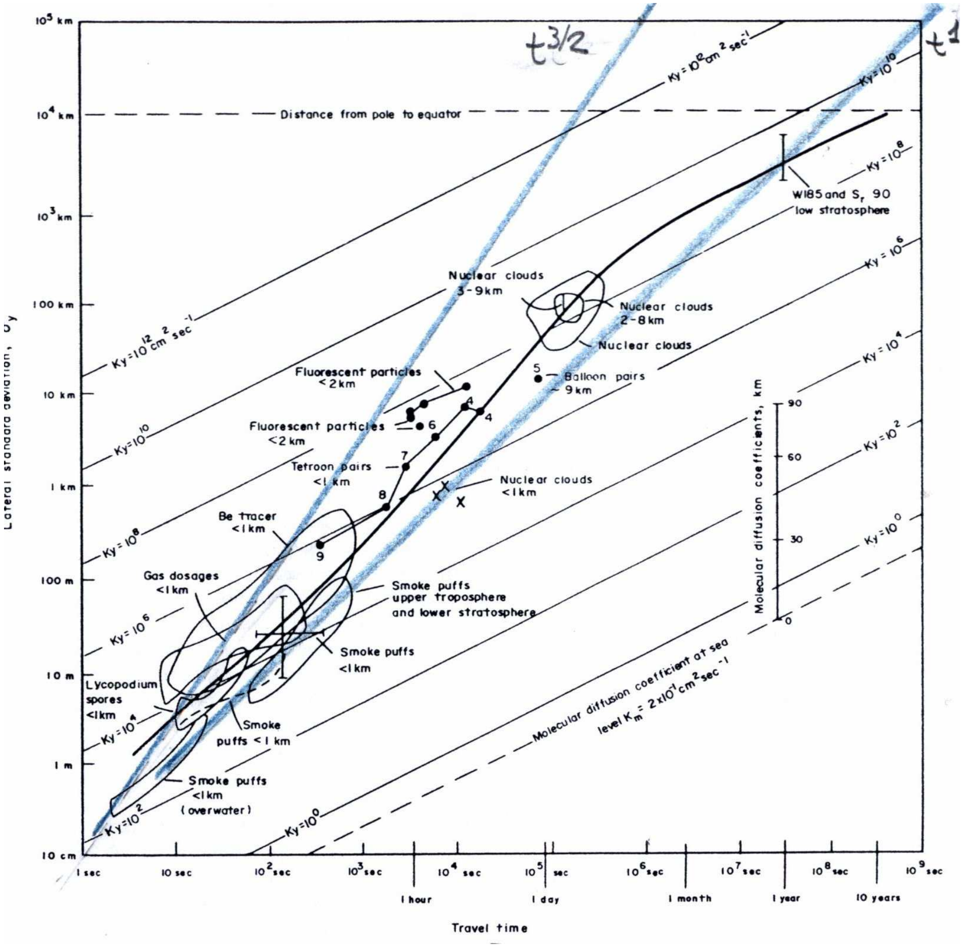
alcuni esempi di traiettorie di palloni rilasciati ogni 10-15 minuti circa, da mezzogiorno del 2 agosto 2000



La dispersione in ambiente urbano



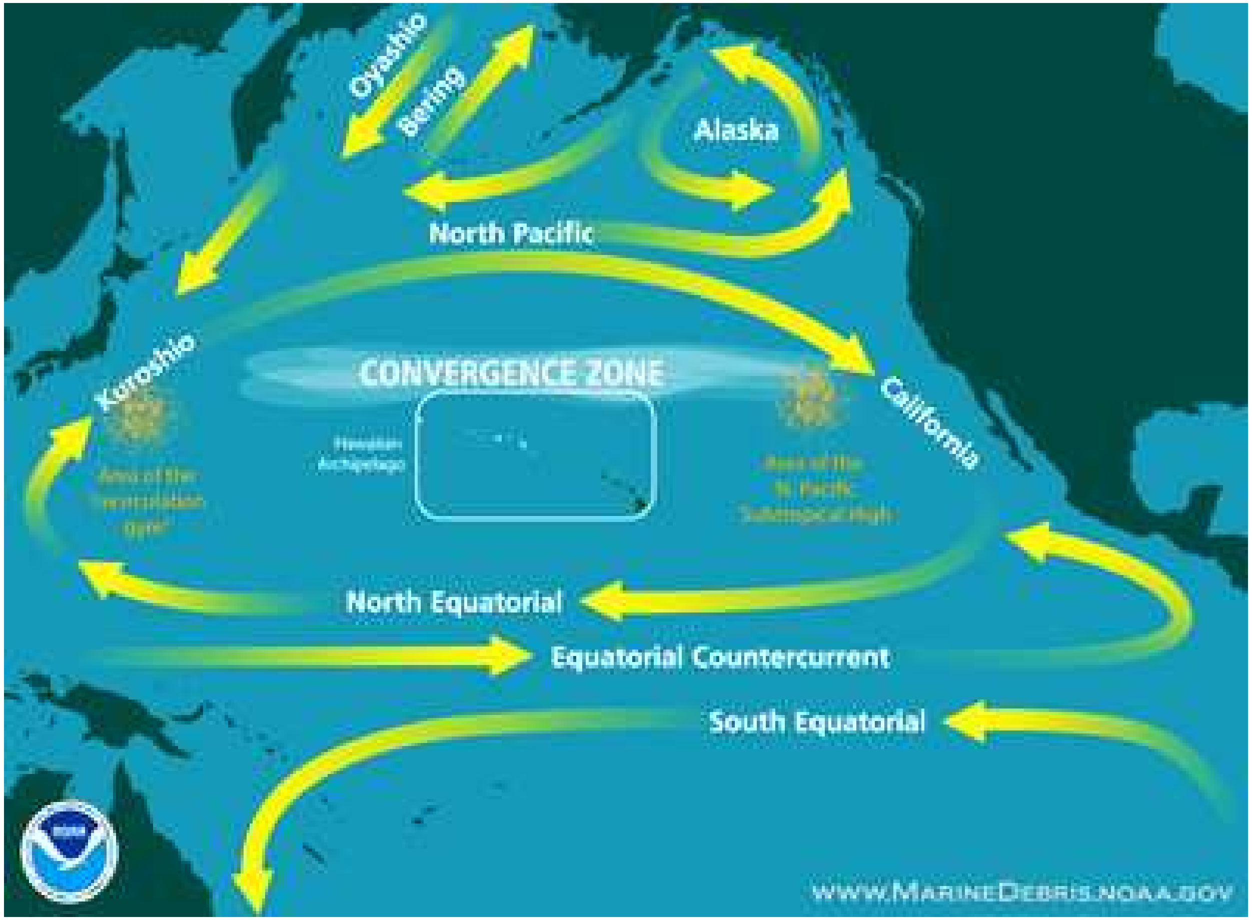
Una sintesi atmosferica, da Gifford (1982)



il coefficiente di diffusione e' funzione del flusso

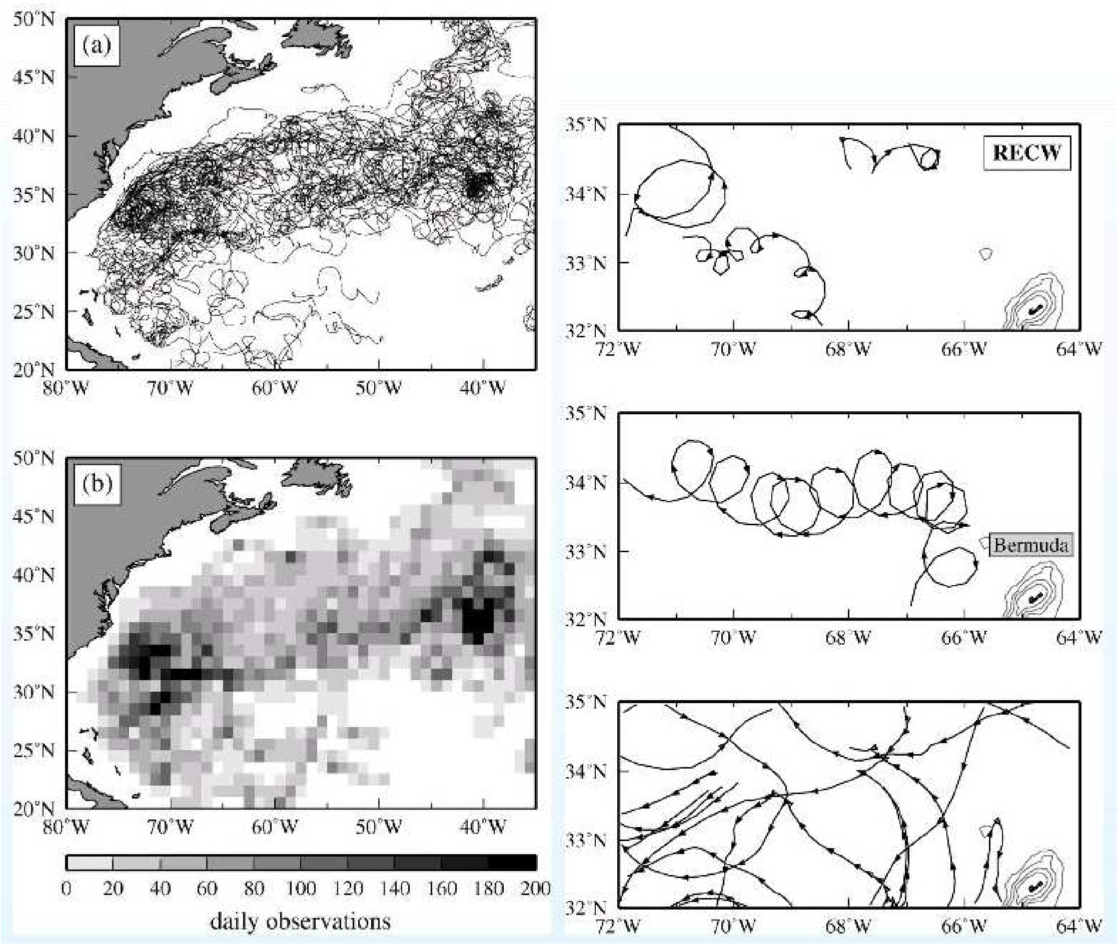
Dispersion of bath toys

On January 10, 1992, 28,800 turtles, ducks, beavers and frogs packed in a cargo container splashed into the mid-Pacific

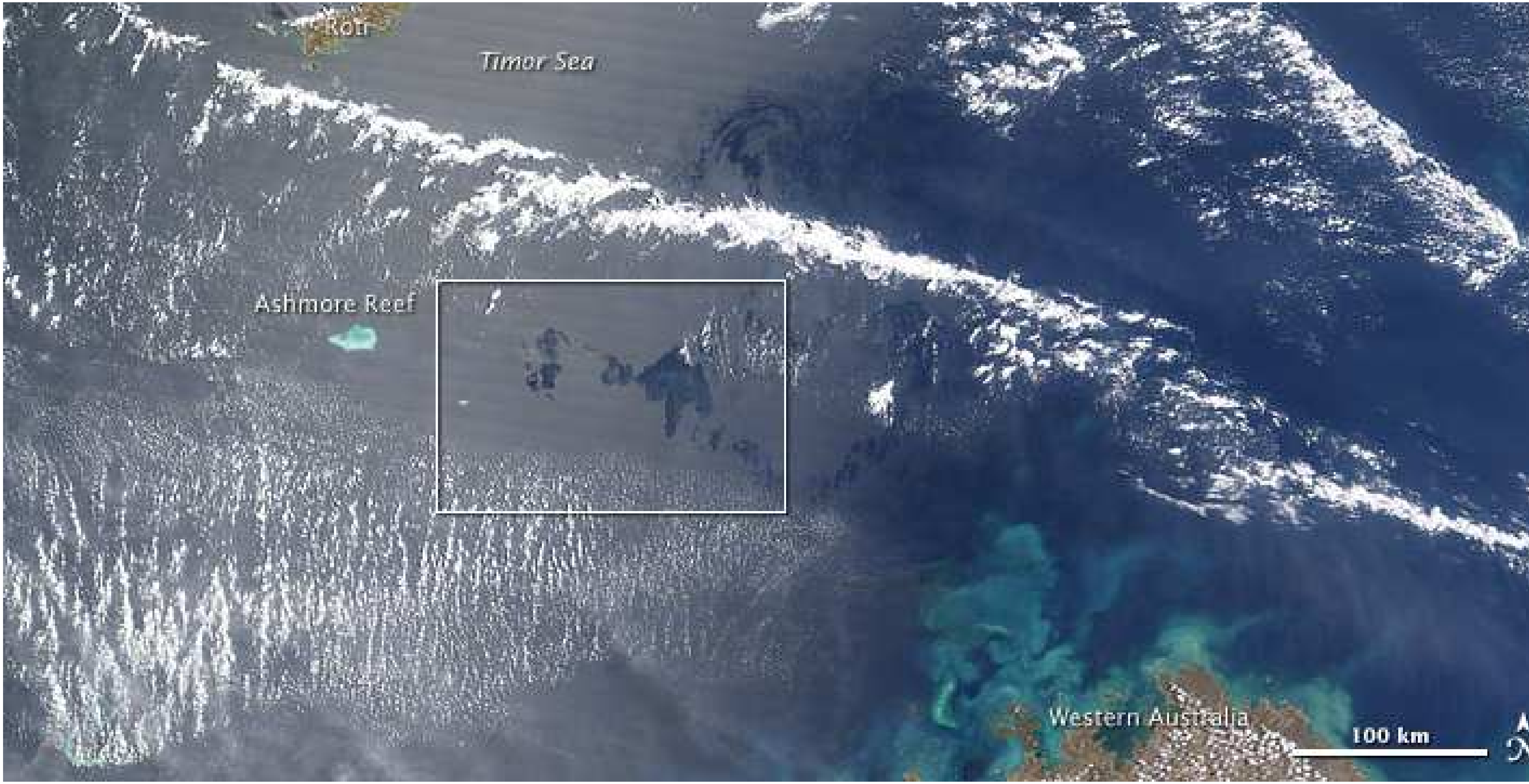


Currents in the Pacific Ocean

Boe in Atlantico, da Veneziani et al. (2004)



problemi di inquinamento marino



perdite di greggio dalle petroliere

Smoke in a village, North Africa

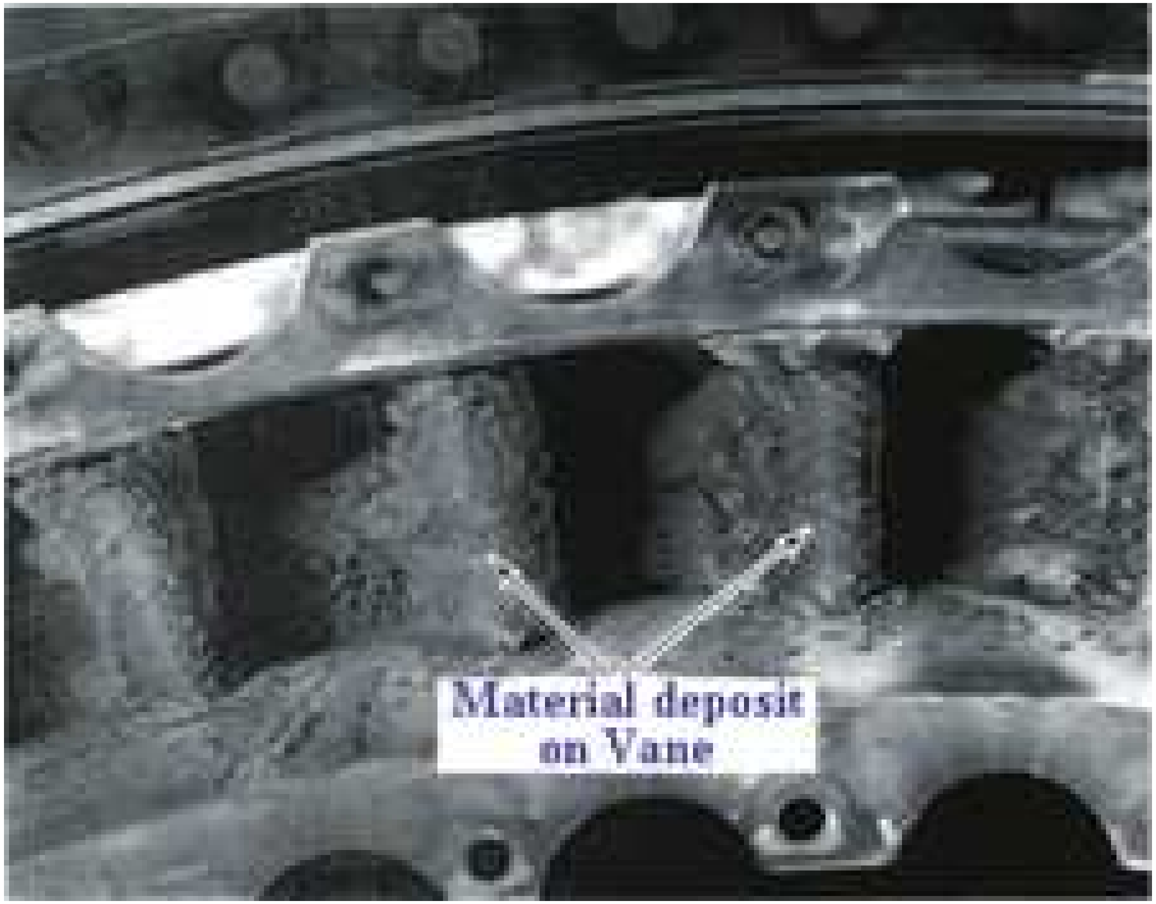
A buoyant plume in a convective boundary layer: note the fluctuations in concentration (courtesy Sandro Finardi, Arianet)

Smoke in the stable marine boundary layer

Smoke released by a ferry in the early morning: observe the apparent absence of fluctuations soon after the emission (courtesy Sandro Finardi, Arianet)

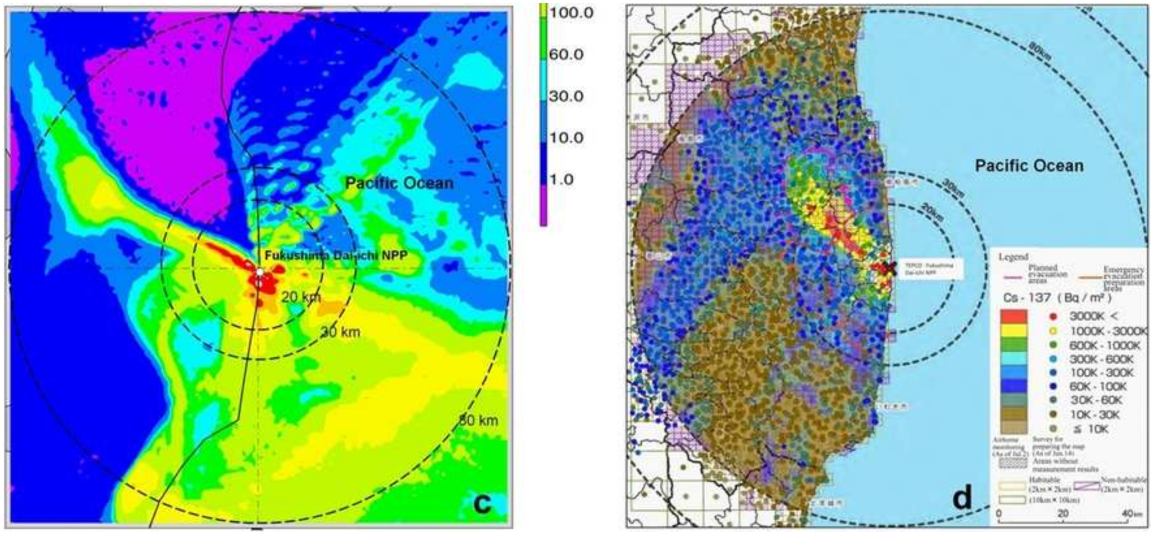
Deposition of ash

Deposition of volcanic-ash material in high-pressure gas turbine vanes: from Guha (2007).



An example of application

Srinivas et al. (2012) computed trajectories at 1h interval to simulate dispersion and deposition of radionuclides emitted by the Dai-Ichi Nuclear Power Plant during the Fukushima accident in the period 11-30 March 2011



Deposition patterns of Cs-137 from simulation (left) and measured values (right)

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